

Problem 4

Q1

A detector runs for $T = 10$ s. The probability of exactly one particle in T is $p = 0.2025$; two or more is impossible. Each 1-second subinterval has the same registration probability. Given no particle in the first nine seconds, find the probability one is registered in the tenth second.

Solution. Let q be the probability of a particle in any one of the ten 1-second intervals. Since at most one particle can occur, these ten events are mutually exclusive, so

$$P(\text{exactly one in } T) = 10q = 0.2025 \Rightarrow q = 0.02025$$

“No particle in the first nine seconds” has probability $1 - 9q$, and the event “particle in the tenth second” is contained in it (only one particle is possible), so its joint probability equals q . Therefore

$$P(10\text{th} \mid \text{none in first } 9) = \frac{q}{1 - 9q} = \frac{0.02025}{0.81775} \approx 0.024763$$

Result: 0.0248

Q2

A keyring has $N = 2025$ keys; exactly one fits. Keys are tried one by one at random without replacement. Find the probability the lock opens on the k -th attempt ($1 \leq k \leq N$).

Solution. The keys are tried in a uniformly random order, so the single correct key is equally likely to occupy any of the N positions. Thus the probability the lock opens on attempt k is the same for every k :

$$P(\text{opens on } k) = \frac{1}{N} = \frac{1}{2025} \approx 0.000494$$

Result: $\frac{1}{2025} \approx 0.0005$ (independent of k)